ARKIV FÖR FYSIK Band 18 nr 17

Communicated 13 April 1960 by G. Borelius

Detonation in a tube with an open space between charge and tube wall

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With 5 figures in the text

In the present paper the detonation of a cylindrical charge lying in an undeformable cylindrical tube is studied theoretically (Fig. 1). Charge and tube are assumed to be co-axial, the charge being initiated at the one end. The charge detonates with a velocity which is assumed to be constant. After passing through the reaction zone the gas usually flows backwards in relation to the front, but in certain cases a flow is observed ahead of the front in the space between charge and tube wall. In the latter case a gas front is generated which moves with a velocity greater than the detonation velocity of the charge.

Symbols

x = distance from end of the tube where the initiation takes place.
t = time, $t=0$ at the moment of initiation.
D = detonation velocity of the charge.
D_g = front velocity of the gas.
F_g = the gas front if $D_g > D$.
$\xi = x/tD$.
$\xi_g = D_g/D$.
p = pressure.
ϱ = density.
$V = 1/\varrho$.
E = internal energy per mass unit.
S = entropy per mass unit.
$c = \sqrt{(dp/d\varrho)_{\text{ad}}}$ sound velocity.
u = particle velocity.
a = cross-sectional area of the charge.
A_1 = internal cross section area of the tube.
$A_2 = A_1 - a$.

Index 0 indicates the intact explosive, index 1 the gas at a cross section L_1 of the tube at a constant distance l_1 behind the reaction zone and index 2 (if $D_g > D$) the gas at a cross section L_2 of the space between charge and tube

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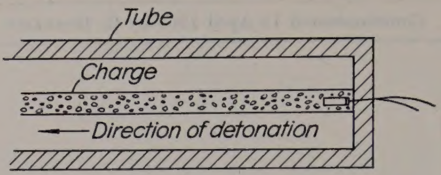

Fig. 1.

wall at a constant distance l_2 ahead of the reaction zone. In L_1 and L_2 velocities, pressures, etc. are assumed to be constant over the entire cross section. Further, we assume that the reaction takes place in the space between L_1 and L_2 and that $dS=0$ after the gas has passed these cross sections. $\bar{D}$ indicates the detonation velocity for the explosive at the charge density $\bar{\varrho}_0=\varrho_0 a/A_1$ according to the Chapman–Jouguet condition. Index g indicates the state at the gas front F_g, and index B the state at the end of the tube where the initiation has taken place.

Fundamental equations

We shall now study the parts of the tube where the entropy remains constant. The time t is assumed to be so large that l_1/Dt and l_2/Dt may be ignored. In such circumstances the state of the gas must be function of ξ and we obtain for example for the pressure [1]

$$dp=\frac{dp}{d\xi}\left(\frac{dx}{Dt}-\frac{x\,dt}{Dt^2}\right)$$

and from this follows $dp=0$ if at least one of the conditions

$$\frac{dp}{d\xi}=0 \quad\text{and}\quad \frac{dx}{dt}=\xi D$$

is fulfilled.

Thus the pressure propagates with a velocity ξD in every point ξ for which $dp/d\xi \neq 0$.

According to Riemann, on the other hand, the quantities

$$\varphi^{+}=u+\int^{p}\frac{dp}{\varrho c} \quad\text{and}\quad \varphi^{-}=u-\int^{p}\frac{dp}{\varrho c}$$

will be propagated at the velocities $u+c$ and $u-c$.

The detonation gas before the reaction zone

Here φ^{+} is constant. The pressure p is consequently either constant in the vicinity of ξ or φ^{-} is propagated at the velocity

$$u-c=\xi D.$$

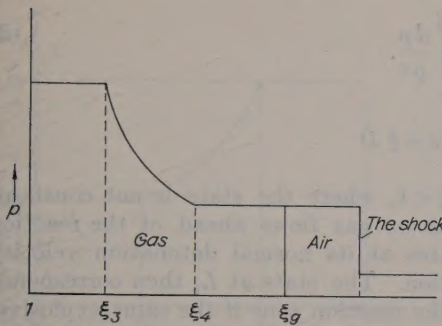

Fig. 2.

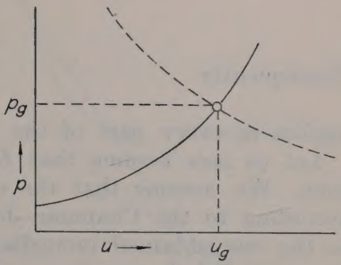

Fig. 3.

If $p_2 > p_g$ the pressure distribution ahead of the front will therefore be the following:

In the interval $1 < \xi \leqslant \xi_3$, in which ξ_3 is determined by the equation

$$u_2 - c_2 = \xi_3 D$$

the pressure p remains constant $= p_2$.

In the interval $\xi_3 \leqslant \xi \leqslant \xi_4$ in which ξ_4 is determined by the equation

$$u_g - c_g = \xi_4 D,$$

the formula $u - c = \xi D$ applies. Here

$$u = u_2 + \int_{p}^{p_2} \frac{dp}{\varrho c}. \tag{1}$$

In the interval $\xi_4 \leqslant \xi \leqslant \xi_g$, finally, p remains constant $= p_g$.

If $p_g = p_2$, then $p = p_g = p_2$ holds good within the entire interval $1 < \xi \leqslant \xi_g$.

Here we have assumed adiabatic expansion. It is clear, however, that if the gas flow generates a shock wave front at ξ' where $1 < \xi' < \xi_g$, then this shock wave will proceed backwards at supersonic velocity until it disappears. $p_2 \geqslant p_g$ therefore always applies. Fig. 2 shows schematically p versus ξ ahead of the front if $p_2 > p_g$.

If we assume the interspace between charge and tube wall filled with air from the beginning, then p_g can be determined with the aid of the Hugoniot curve of the air. In Fig. 3 the unbroken curve indicates pressure versus piston velocity for shock waves in air. The point of intersection between this curve and the curve of dashes representing eq. (1) gives the point p_g, u_g.

The detonation gas behind the reaction zone

Let us now proceed to the section $0 < \xi < 1$ behind the reaction zone. As we have seen above, the pressure will propagate with the velocity ξD in every point ξ for which $dp/d\xi \neq 0$. On the other hand, the propagating velocity is according to Riemann equal to $u + c$, where u is determined by the formula

$$u = u_1 + \int_{p_1}^{p} \frac{dp}{\varrho c}. \tag{2}$$

Consequently
$$u + c = \xi D$$

applies in every part of the interval $0 < \xi < 1$, where the state is not constant.

Let us now assume that $D_g = D$, so that no gas flows ahead of the reaction zone. We assume that the charge detonates at its normal detonation velocity according to the Chapman–Jouguet condition. The state at L_1 then corresponds to the one obtained immediately behind the reaction zone if the same explosive, but with the density $\bar{\varrho}_0 = \varrho_0 a/A_1$ detonates at the detonation velocity D. The detonation velocity corresponding to the density $\bar{\varrho}_0$ according to Chapman–Jouguet is $< D$, i.e. the detonation velocity D is in reality supernormal [2]. Behind the detonation front a section with a constant pressure extends from the front $\xi = 1$ to ξ_5 determined by the equation

$$u_1 + c_1 = \xi_5 D.$$

Behind ξ_5 the pressure falls monotonously with decreasing ξ according to the formula

$$u + c = \xi D$$

in which u is determined by (2). This takes place until u or p attains the boundary condition which applies at $\xi = 0$. If this occurs in the point ξ_6 then $u = u_B$, $p = p_B$, etc. applies for $0 \leqslant \xi \leqslant \xi_6$. Fig. 4 shows schematically pressure versus ξ when the tube is closed at the end $x = 0$.

In the Hugoniot diagram Fig. 5 for the explosive of density $\bar{\varrho}_0 = \varrho_0 a/A_1$ the detonation point with pressure p_1 and spec. volume V_1 will be represented by the point A. The straight line from the initial point to A corresponds to the detonation velocity

$$D = \frac{1}{\bar{\varrho}_0} \sqrt{\frac{p_1}{1/\bar{\varrho}_0 - V_1}}.$$

Conditions for $D_g > D$

If the charge detonates at its normal velocity according to Chapman–Jouguet, D_g will be $= D$, i.e. the detonation gas will not flow ahead of the detonation front. This holds good as long as $D > \bar{D}$. But if $D < \bar{D}$ this corresponds in the Hugoniot diagram (Fig. 5) to the detonation point lying on the line of dashes which does not cut the Hugoniot curve. The only possible explanation is that a part of the detonation gas flows forward ahead of the detonation front.

This phenomenon is observed when dynamite detonates at a low velocity. For 25 mm cartridges this is ca. 2.300 m/s, whilst $\bar{D}$ possibly falls below this value only for very small values of a/A_1. The detonation process has been studied in various tubes at the Nitroglycerin AB, Physical Research Laboratory [3]. It has been found that the detonation fades out in certain tubes, probably due

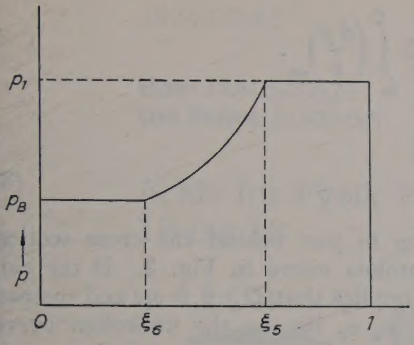

Fig. 4.

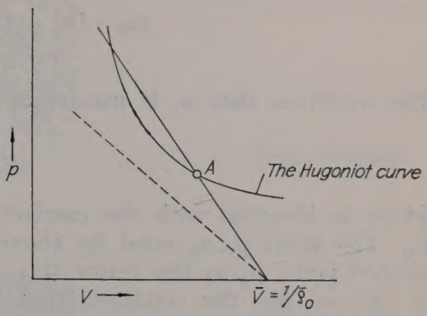

Fig. 5.

to the dynamite being dead-pressed by the compressed air and gas layer ahead of the detonation front.

The question if a stable, stationary detonation process can develop in an undeformable tube if $D < \bar{D}$ is probably merely of theoretical interest. We simplify the problem essentially by assuming that the charge lies protected against the pressure from the gas and air layer ahead of the front. Let the released energy per mass unit of the detonation gas which flows through the cross-section L_1 and/or L_2 be Q_1 and/or Q_2 and let n_1 and n_2 be the fraction of the explosive which flows through these cross sections. The following equations then apply

$$n_1 + n_2 = 1,$$

$$n_1 \varrho_0 a D = \varrho_1 A_1 (D - u_1),$$

$$n_2 \varrho_0 a D = \varrho_2 A_2 (u_2 - D),$$

$$D = n_1 \left((D - u_1) + \frac{p_1}{\varrho_1 (D - u_1)} \right) - n_2 \left((u_2 - D) + \frac{p_2}{\varrho_2 (u_2 - D)} \right),$$

$$\frac{D^2}{2} = n_1 \left(\frac{(D - u_1)^2}{2} + \Delta E_1 + \frac{p_1}{\varrho_1} - Q_1 \right) + n_2 \left(\frac{(u_2 - D)^2}{2} + \Delta E_2 + \frac{p_2}{\varrho_2} - Q_2 \right).$$

The first three equations express the constancy of the mass, the fourth that of the impulse and the last that of the energy. To this shall be added two equations of state:

$$E_1 = F_1 (p_1, \varrho_1), \quad E_2 = F_2 (p_2, \varrho_2).$$

We thus have in all 7 equations and 10 unknown quantities. Of the missing equations, one at least must express a property of the reaction. The simplest suggestion is that the gas which flows through the two cross sections L_1 and L_2 originates from the same reaction zone and expands adiabatically after leaving it. This gives

$$S_1 = S_2,$$

$$(u_2-D)^2-(D-u_1)^2=2\int_{p_2}^{p_1}\left(\frac{dp}{\varrho}\right)_{\text{ad}}.$$

The condition that n_1 is maximum gives

$$D-u_1=c_1, \tag{3}$$

which is identical with the rarefaction setting in just behind the cross section L_1. The point p_2, u_2 must lie above the unbroken curve in Fig. 3. If the calculated point p_2, u_2 lies below this curve, it implies that $D-u_1>c_1$, and instead of (3) we get the condition that the point p_2, u_2 lies on the unbroken curve in Fig. 3.

A numerical calculation of the possible ways of propagation according to the principle described would probably be rather comprehensive and for various reasons of no great practical interest.

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Tryckt den 23 september 1960

Uppsala 1960. Almqvist & Wiksells Boktryckeri AB